

# Backward charmonium production in $\pi N$ collisions

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## Abstract:

The QCD collinear factorization framework allows to describe exclusive backward production of a  $J/\psi$  meson in pion-nucleon collisions in terms of pion-to-nucleon transition distribution amplitudes. We calculate the scattering amplitude at the leading order in the strong coupling constant and estimate the cross section of this reaction in the backward kinematical region for a medium energy pion beam available at the J-Parc experimental facility.

# 1 Introduction

Besides leptonproduction experiments, hadronic facilities open complementary accesses to the study of the partonic content of hadrons. Indeed the collinear factorization theorems of quantum chromodynamics (QCD) allow to define universal hadronic matrix elements which enter scattering amplitudes in both lepton-nucleon and meson-nucleon reactions in specific kinematical conditions. This statement is true for both inclusive and exclusive reactions. In the last two decades, we have witnessed a tremendous progress in the understanding of deeply virtual Compton scattering (DVCS) and deep meson electroproduction within this framework. The detailed study of generalized parton distributions (GPDs) [1], the relevant hadronic matrix elements, is a major goal of modern hadronic physics. Timelike processes such as the timelike Compton scattering (TCS) [2] and exclusive Drell-Yan production in  $\pi N$  collisions [3] obey the same factorization properties and allow to access the same GPDs. Exclusive charmonium production has also been addressed in the same framework [4].

The extension of the collinear factorization approach to other processes such as backward virtual Compton scattering and backward meson electroproduction, has been advocated [5, 6, 7] – although the corresponding factorization theorems have not yet been rigorously proven. This lead to the definition of new hadronic matrix elements of three quark operators on the light cone, the nucleon-to-meson transition distribution amplitudes (TDAs) [8]. To motivate the validity of such a factorization regime, one requires the existence of a large scale  $Q$ , which may be taken as the virtuality of the photon quantifying the electromagnetic probe or the mass of a heavy quark in the case of heavy quarkonium production. This large scale plays the role of the factorization scale and determines the magnitude of the QCD coupling constant  $\alpha_s$ .

The intense pion beam available at the Japan Proton Accelerator Research Complex (J-Parc) opens the possibility to study hard exclusive reactions such as lepton pair or charmonium production in  $\pi N$  collisions. This will provide new ways of testing the universality of GPDs and TDAs. The recent feasibility study [9] of forward lepton pair production suggests that GPDs can be accessed there. Here we address the complementary case of backward charmonium production, the perturbative QCD description of which involves pion-to-nucleon TDAs. This process can be seen as the cross-channel counterpart of nucleon-antinucleon annihilation into heavy quarkonium in association with a pion. The description of this latter process within the collinear factorization approach involving nucleon-to-pion TDAs was studied in Ref. [10]. The cross-section estimates performed for the kinematical conditions of  $\bar{\text{P}}\text{ANDA@GSI-FAIR}$  lead to large enough production rates to be experimentally accessed [11, 12].

## 2 Kinematics of the reaction

In the present paper we consider the reaction

$$\pi^-(p_\pi) + N^p(p_1) \rightarrow J/\psi(p_\psi) + N^n(p_2). \quad (1)$$

The  $\pi N$  center-of-mass energy squared  $s = (p_\pi + p_1)^2 \equiv W^2$  and the charmonium mass squared  $M_\psi^2$  introduce the natural hard scale. In complete analogy with our analysis of the nucleon-antinucleon annihilation process [13, 14] we assume that this reaction admits a factorized description in the near-backward kinematical regime (see Fig. 1), where  $|u| \equiv |\Delta^2| = |(p_2 - p_\pi)^2| \ll W^2, M_\psi^2$ . This corresponds to the final nucleon moving almost in the direction of the initial pion in  $\pi N$  center-of-mass system (CMS).

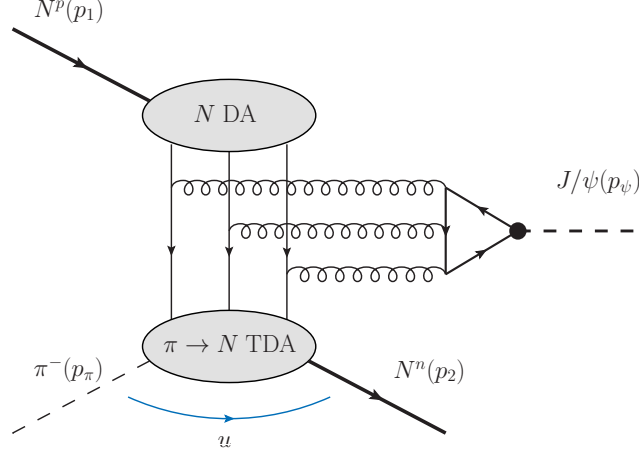


Figure 1: Collinear factorization of the  $\pi^-(p_\pi) + N^p(p_1) \rightarrow N^n(p_2) + J/\psi(p_\psi)$  reaction in the  $u$ -channel regime. DA stands for the distribution amplitude of the incoming nucleon;  $\pi \rightarrow N$  TDA stands for the transition distribution amplitude from a pion to a nucleon.

The  $z$ -axis is chosen along the direction of the pion beam in the meson-nucleon CMS frame. We introduce the light-cone vectors  $p, n$  satisfying  $2p \cdot n = 1$ . The Sudakov decomposition of the relevant momenta reads

$$\begin{aligned}
 p_\pi &= (1 + \xi)p + \frac{m_\pi^2}{1 + \xi}n; \\
 p_1 &= \frac{2(1 + \xi)m_N^2}{W^2 + \Lambda(W^2, m_N^2, m_\pi^2) - m_N^2 - m_\pi^2}p + \frac{W^2 + \Lambda(W^2, m_N^2, m_\pi^2) - m_N^2 - m_\pi^2}{2(1 + \xi)}n; \\
 \Delta \equiv (p_2 - p_\pi) &= -2\xi p + \left( \frac{m_N^2 - \Delta_T^2}{1 - \xi} - \frac{m_\pi^2}{1 + \xi} \right) n + \Delta_T; \\
 p_\psi &= p_1 - \Delta; \quad p_2 = p_\pi + \Delta,
 \end{aligned} \tag{2}$$

where

$$\Lambda(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2xy - 2xz - 2yz} \tag{3}$$

is the Mandelstam function and  $m_N$  and  $m_\pi$  stand respectively for the nucleon and pion masses. The transverse direction in (2) is defined with respect to the  $z$  direction and  $\xi$  is the skewness variable

$$\xi \equiv -\frac{(p_2 - p_\pi) \cdot n}{(p_2 + p_\pi) \cdot n}. \tag{4}$$

Within the collinear factorization framework we neglect both the pion and nucleon masses with respect to  $M_\psi$  and  $W$  and set  $\Delta_T = 0$  within the coefficient function. This results in the approximate expression for the skewness variable (4):

$$\xi \simeq \frac{M_\psi^2}{2W^2 - M_\psi^2}. \quad (5)$$

However, the approximation (5) can potentially affect the definition of the physical domain of the reaction (1) determined by the requirement  $\Delta_T^2 \leq 0$ , where

$$\Delta_T^2 = \frac{1 - \xi}{1 + \xi} \left( u - 2\xi \left[ \frac{m_\pi^2}{1 + \xi} - \frac{m_N^2}{1 - \xi} \right] \right). \quad (6)$$

To improve the approximate kinematical formulas in the vicinity of the threshold it is sometimes convenient to keep partly the finite mass corrections resulting in the modified expression for skewness variable

$$\xi = \frac{M_\psi^2 - m_N^2 - u}{W^2 + \Lambda(W^2, m_N^2, m_\pi^2) + u - M_\psi^2 - m_\pi^2} + O\left(\frac{m_N^4}{W^4}\right) + O\left(\frac{m_N^2 u}{W^4}\right) + O\left(\frac{m_N^2 m_\pi^2}{W^4}\right). \quad (7)$$

In order to control the validity of the kinematic approximations (5), (7) it is instructive to consider the exact kinematics of the reaction (1) in the  $\pi N$  CMS frame. In this frame the relevant momenta read:

$$\begin{aligned} p_\pi &= \left( \frac{W^2 + m_\pi^2 - m_N^2}{2W}, \vec{p}_\pi \right); & p_\psi &= \left( \frac{W^2 + M_\psi^2 - m_N^2}{2W}, -\vec{p}_2 \right); \\ p_1 &= \left( \frac{W^2 + m_N^2 - m_\pi^2}{2W}, -\vec{p}_\pi \right); & p_2 &= \left( \frac{W^2 + m_N^2 - M_\psi^2}{2W}, \vec{p}_2 \right), \end{aligned} \quad (8)$$

where

$$|\vec{p}_\pi| = \frac{\Lambda(W^2, m_N^2, m_\pi^2)}{2W}; \quad |\vec{p}_2| = \frac{\Lambda(W^2, m_N^2, M_\psi^2)}{2W}. \quad (9)$$

The CMS scattering angle  $\theta_u^*$  is defined as the angle between  $\vec{p}_\pi$  and  $\vec{p}_2$ :

$$\cos \theta_u^* = \frac{2W^2(u - m_N^2 - m_\pi^2) + (W^2 + m_\pi^2 - m_N^2)(W^2 + m_N^2 - M_\psi^2)}{\Lambda(W^2, m_N^2, m_\pi^2)\Lambda(W^2, m_N^2, M_\psi^2)}. \quad (10)$$

The transverse momentum transfer squared (6) is then given by

$$\Delta_T^2 = -\frac{\Lambda^2(W^2, M_\psi^2, m_N^2)}{4W^2}(1 - \cos^2 \theta_u^*) \quad (11)$$

and the physical domain for the reaction (1) is defined from the requirement that  $\Delta_T^2 \leq 0$ .

- In particular, the backward kinematics regime  $\theta_u^* = 0$  corresponds to  $\vec{p}_2$  along  $\vec{p}_\pi$ , which means that  $J/\psi$  is produced along  $-\vec{p}_\pi$  *i.e.* in the backward direction. In this case  $u$  reaches its maximal value

$$\begin{aligned} u_{\max} &\equiv \frac{2\xi(m_\pi^2(\xi - 1) + m_N^2(\xi + 1))}{\xi^2 - 1} \\ &= m_N^2 + m_\pi^2 - \frac{(W^2 + m_\pi^2 - m_N^2)(W^2 + m_N^2 - M_\psi^2)}{2W^2} + 2|\vec{p}_\pi||\vec{p}_\psi|. \end{aligned} \quad (12)$$

At the same time  $t = (p_2 - p_1)^2$  reaches its minimal value  $t_{\min}$  ( $W^2 + u_{\max} + t_{\min} = 2m_N^2 + m_\pi^2 + M_\psi^2$ ).

Note that  $u$  is negative, and therefore  $|u_{\max}|$  is the minimal possible absolute value of the momentum transfer squared. It is for  $u \sim u_{\max}$  that one may expect to satisfy the requirement  $|u| \ll W^2, M_\psi^2$  which is crucial for the validity of the factorized description of (1) in terms of  $\pi \rightarrow N$  TDAs and nucleon DAs.

- Another limiting value  $\theta_u^* = \pi$  corresponds to  $\vec{p}_2$  along  $-\vec{p}_\pi$  *i.e.*  $J/\psi$  produced in the forward direction. In this case  $u$  reaches its minimal value

$$u_{\min} = m_N^2 + m_\pi^2 - \frac{(W^2 + m_\pi^2 - m_N^2)(W^2 + m_N^2 - M_\psi^2)}{2W^2} - 2|\vec{p}_\pi||\vec{p}_\psi|. \quad (13)$$

At the same time  $t = (p_2 - p_1)^2$  reaches its maximal value  $t_{\max}$ . The factorized description in terms of  $\pi \rightarrow N$  TDAs does not apply in this case as  $|u|$  turns to be of order of  $W^2$ .

### 3 Hard part of the $\pi^- + N^p \rightarrow J/\psi + N^n$ amplitude

The calculation of  $\pi^- + N^p \rightarrow J/\psi + N^n$  scattering amplitude follows the same main steps as the classical calculation of the  $J/\psi \rightarrow p + \bar{p}$  decay amplitude [15, 16, 17, 18]. Assuming the factorization of small and large distance dynamics the hard part of the amplitude is computed within perturbative QCD (pQCD). Large distance dynamics is encoded within the matrix elements of QCD light-cone operators between the appropriate hadronic states.

The leading order amplitude of the  $J/\psi N^n$  production subprocess of (1) is, up to the reverse of direction of fermionic lines, given by the sum of the same three diagrams presented on Figure 2 of Ref. [10].

For the calculation of these diagrams we apply the collinear approximation, neglecting both the nucleon and pion masses and assuming  $\Delta_T = 0$ . Therefore, the Sudakov decomposition (2) reads as:

$$p_\pi \simeq (1 + \xi)p; \quad p_1 \simeq \frac{s}{(1 + \xi)}n; \quad p_\psi \simeq 2\xi p + \frac{s}{(1 + \xi)}n; \quad \Delta \simeq -2\xi p. \quad (14)$$

Also throughout our calculation we set  $M_\psi \simeq 2m_c \equiv \bar{M}$  with  $\bar{M} = 3 \text{ GeV}$ .

Below we summarize our conventions for the relevant light-cone matrix elements encoding the soft dynamics.

- The leading twist-3  $uud$   $\pi^-$ -to-neutron ( $\pi^- \rightarrow n$ ) TDAs are defined from the Fourier transform

$$\mathcal{F} \equiv (p \cdot n)^3 \int \left[ \prod_{j=1}^3 \frac{d\lambda_j}{2\pi} \right] e^{i \sum_{k=1}^3 x_k \lambda_k (p \cdot n)} \quad (15)$$

of the  $n\pi^-$  matrix element of the trilinear antiquark operator on the light cone

$$M_{\rho\tau\chi}^{(\pi^-\rightarrow n)}(\lambda_1 n, \lambda_2 n, \lambda_3 n) = \langle n(p_2) | \varepsilon_{c_1 c_2 c_3} \bar{u}_\rho^{c_1}(\lambda_1 n) \bar{u}_\tau^{c_2}(\lambda_2 n) \bar{d}_\chi^{c_3}(\lambda_3 n) | \pi^-(p_\pi) \rangle. \quad (16)$$

Namely,

$$\begin{aligned} & 4\mathcal{F}M_{\rho\tau\chi}^{(\pi^-\rightarrow n)}(\lambda_1 n, \lambda_2 n, \lambda_3 n) \\ &= \delta(x_1 + x_2 + x_3 - 2\xi) i \frac{f_N}{f_\pi} \sum_{\substack{\text{Dirac} \\ \text{structures}}} s_{\rho\tau, \chi}^{(\pi \rightarrow N)} H_s^{(\pi^-\rightarrow n)}(x_1, x_2, x_3, \xi, \Delta^2), \end{aligned} \quad (17)$$

where the sum goes over the eight leading twist Dirac structures  $s_{\rho\tau, \chi}^{(\pi \rightarrow N)}$  built of  $p$ ,  $n$ ,  $\Delta_T$ , charge conjugation matrix  $C$  and the Dirac spinor  $\bar{U}(p_2)$ . The explicit form of these Dirac structures is worked out in the Appendix A, which contains also the relation of the parametrization of the leading twist  $\pi^- \rightarrow n$  TDAs to the conventional  $n \rightarrow \pi^-$  TDAs introduced in Ref. [19].

- For the leading twist antinucleon DAs we employ the standard parametrization of Ref. [17] involving three invariant functions  $V^p$ ,  $A^p$  and  $T^p$  (see also Appendix B of Ref. [14]).
- For the light-cone wave function of  $J/\psi$  heavy quarkonium we use the so-called non-relativistic wave function suggested in Ref. [17].

The leading order and leading twist amplitude of the reaction (1) admits the following parametrization<sup>1</sup>

$$\begin{aligned} \mathcal{M}_\lambda^{s_1 s_2} &= \mathcal{C} \frac{1}{M^5} \left[ \bar{U}(p_2, s_2) \hat{\mathcal{E}}^*(\lambda) \gamma_5 U(p_1, s_1) \mathcal{I}(\xi, \Delta^2) \right. \\ &\quad \left. - \frac{1}{m_N} \bar{U}(p_2, s_2) \hat{\mathcal{E}}^*(\lambda) \hat{\Delta}_T \gamma_5 U(p_1, s_1) \mathcal{I}'(\xi, \Delta^2) \right], \end{aligned} \quad (18)$$

where  $\mathcal{E}$  is the charmonium polarization vector and  $\bar{U}$ ,  $U$  stand for the nucleon Dirac spinors.

The calculation of the 3 Born order diagrams yields the same result for the integral convolutions  $\{\mathcal{I}, \mathcal{I}'\}(\xi, \Delta^2)$  as for  $J/\psi$   $\pi$  production in  $\bar{p}p$  annihilation up to the obvious replacement of nucleon-to-pion ( $N \rightarrow \pi$ ) TDAs with pion-to-nucleon ( $\pi \rightarrow N$ ) TDAs introduced in (17). The explicit expressions for  $\mathcal{I}$ ,  $\mathcal{I}'$  can be found in eqs. (13), (15) of Ref. [10]. The overall numerical factor  $\mathcal{C}$  in (18) is expressed as:

$$\mathcal{C} = (4\pi\alpha_s)^3 \frac{f_N^2 f_\psi}{f_\pi} \frac{10}{81}, \quad (19)$$

where  $\alpha_s$  stands for the strong coupling,  $f_\pi = 93$  MeV is the pion weak decay constant,  $f_\psi$  determines the normalization of the wave function of heavy quarkonium and  $f_N$  determines the value of the nucleon wave function at the origin. The normalization constant  $f_\psi$  is extracted from the charmonium leptonic decay width  $\Gamma(J/\psi \rightarrow e^+ e^-)$ . With the values quoted in [20] we get  $|f_\psi| = 416 \pm 5$  MeV.

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<sup>1</sup>We adopt Dirac's "hat" notation  $\hat{v} \equiv v_\mu \gamma^\mu$ .

## 4 Estimates of the cross section

To work out the cross section formula we square the amplitude (18) and average (sum) over spins of initial (final) nucleons. Staying at the leading twist accuracy we account for the production of transversely polarized  $J/\psi$ . Summing over the transverse polarizations we find

$$|\overline{\mathcal{M}}_T|^2 = \sum_{\lambda_T} \left( \frac{1}{2} \sum_{s_1 s_2} \mathcal{M}_{\lambda_T}^{s_1 s_2} (\mathcal{M}_{\lambda_T}^{s_1 s_2})^* \right). \quad (20)$$

The leading twist differential cross section of  $\pi + N \rightarrow J/\psi + N$  then reads

$$\begin{aligned} \frac{d\sigma}{d\Delta^2} &= \frac{1}{16\pi\Lambda^2(s, m_N^2, m_\pi^2)} |\overline{\mathcal{M}}_T|^2 \\ &= \frac{1}{16\pi\Lambda^2(s, m_N^2, m_\pi^2)} \frac{1}{2} |\mathcal{C}|^2 \frac{2(1+\xi)}{\xi \bar{M}^8} \left( |\mathcal{I}(\xi, \Delta^2)|^2 - \frac{\Delta_T^2}{m_N^2} |\mathcal{I}'(\xi, \Delta^2)|^2 \right), \end{aligned} \quad (21)$$

where  $\Lambda(x, y, z)$  is defined in (3).

In order to get a rough estimate of the cross section we use the simple nucleon exchange model for  $\pi \rightarrow N$  TDAs suggested in [22]. We do not expect that the inclusion of the spectral part for  $\pi \rightarrow N$  TDAs [21, 23] would be essential to draw a conclusion on the feasibility of the relevant experiment. The refinement of the present description will be done in course of availability of precise experimental data.

The predictions of the cross-channel nucleon exchange model of Ref. [22] for  $n \rightarrow \pi^-$  TDAs within the parametrization (A1) are summarized in eqs. (25)–(27) of Ref. [10]. Employing the results of the Appendix A we conclude that  $\pi^- \rightarrow n$  TDAs within this model are expressed as

$$\begin{aligned} &\{V_{1,2}, A_{1,2}, T_{1,2,3}\}^{(\pi^- \rightarrow n)}(x_{1,2,3}, \xi, \Delta^2) \Big|_{N(940)} \\ &= \sqrt{2} \{V_{1,2}, A_{1,2}, T_{1,2,3}\}^{(p \rightarrow \pi^0)}(-x_{1,2,3}, -\xi, \Delta^2) \Big|_{N(940)}; \\ &T_4^{(\pi^- \rightarrow n)}(x_{1,2,3}, \xi, \Delta^2) \Big|_{N(940)} = 0. \end{aligned} \quad (22)$$

Note that nucleon-to-pion TDAs within the cross channel nucleon exchange model have purely the Efremov-Radyushkin-Brodsky-Lepage (ERBL)-like support. As the result the convolution integrals  $\mathcal{I}, \mathcal{I}'$  within this model turn to be real since the poles in the corresponding integrands are located either on the cross-over trajectories which separate the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP)-like and the ERBL-like support regions of  $\pi \rightarrow N$  TDAs<sup>2</sup>, or within the DGLAP-like support regions.

The convolution integrals  $\mathcal{I}, \mathcal{I}'$  within the simple nucleon pole model (22) are ex-

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<sup>2</sup>For the definition of the ERBL-like and DGLAP-like support regions of TDAs see [21].

pressed as

$$\begin{aligned}\mathcal{I}(\xi, \Delta^2)\Big|_{N(940)} &= -\sqrt{2}\frac{f_\pi g_{\pi NN} m_N (1+\xi)}{(\Delta^2 - m_N^2)(1-\xi)} M_0; \\ \mathcal{I}'(\xi, \Delta^2)\Big|_{N(940)} &= -\sqrt{2}\frac{f_\pi g_{\pi NN} m_N}{(\Delta^2 - m_N^2)} M_0,\end{aligned}\tag{23}$$

where  $M_0$  is given by eq. (19) of [10]. Note that the integral convolution  $M_0$  also occurs in the well-known expression for the  $J/\psi \rightarrow \bar{p}p$  decay width within the pQCD approach [18].

As the phenomenological input for the cross section estimate we may employ different solutions for the leading twist nucleon DAs  $V^p$ ,  $A^p$ ,  $T^p$ . Similarly to the case of charmonium decay width, our result depends strongly both on the form of the input leading twist nucleon DAs and the value of  $\alpha_s$ .

Analogously to Ref. [10], we have chosen to present our results for the  $\pi^- p \rightarrow J/\psi n$  cross section with the value of  $\alpha_s$  fixed by the requirement that the given phenomenological solution reproduces the experimental  $J/\psi \rightarrow N\bar{N}$  decay width from the pQCD expression of Ref. [18]. The corresponding values of  $\alpha_s$  for several phenomenological solutions for the leading twist nucleon DA are summarized in the Table 1 of Ref. [10].

Phenomenological solutions for nucleon DAs that are largely concentrated in the end-point regions such as the Chernyak-Ogloblin-Zhitnitsky (COZ) [18] or King-Sachrajda (KS) [24] require smaller values of  $\alpha_s \sim 0.25$  to reproduce the experimental value of  $\Gamma(J/\psi \rightarrow p\bar{p})$ . The solutions which are close to the asymptotic form of the nucleon DA  $\phi_{\text{as}}^N(y_{1,2,3}) \equiv V_{\text{as}}^p(y_{1,2,3}) - A_{\text{as}}^p(y_{1,2,3}) = 120y_1y_2y_3$  like the Bolz-Kroll (BK) [25] and Braun-Lenz-Wittmann next-to-leading order (BLW NLO) model [26] require rather large values of  $\alpha_s \sim 0.4$  to reproduce the experimental value of  $\Gamma(J/\psi \rightarrow p\bar{p})$ . We refer the reader to Refs. [27, 28] for the discussion and critics of the available phenomenological models of the leading twist nucleon DAs.

On Fig. 2 we show our estimates of the differential cross section  $\frac{d\sigma}{d\Delta^2}$  for  $\pi^- p \rightarrow J/\psi n$  (21) as a function of the pion beam momentum  $P_\pi$  ( $W^2 = m_N^2 + m_\pi^2 + 2E_\pi m_N \approx 2m_N P_\pi$ ) for the exactly backward charmonium production ( $\Delta_T^2 = 0 \Leftrightarrow \theta_\pi^* = 0$ ). The range of the pion beam momentum  $10 \text{ GeV} \leq P_\pi \leq 20 \text{ GeV}$  corresponds to the J-Parc experimental setup.

On Fig. 3 we show the differential cross section  $\frac{d\sigma}{d\Delta^2}$  for  $\pi^- p \rightarrow J/\psi n$  as a function of  $|u - u_{\text{max}}|$  for  $|u| \leq 1 \text{ GeV}^2$ , where  $u_{\text{max}}$  is the threshold value (12) of the momentum transfer squared. We show the result for several values of the pion beam energy  $P_\pi$ . In order to better describe the cross section behavior for  $\Delta_T^2 \neq 0$  we employ the exact value (12) for the maximal cross channel momentum transfer squared.



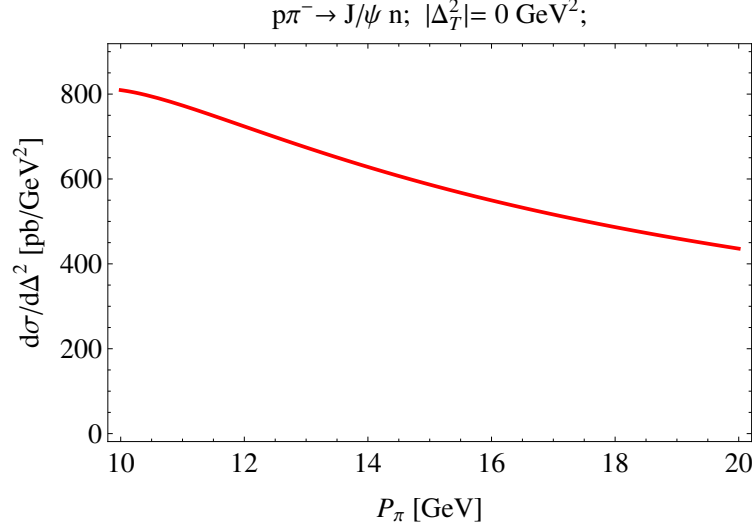


Figure 2: Differential cross section  $\frac{d\sigma}{d\Delta^2}$  for  $\pi^- p \rightarrow J/\psi n$  as a function of the pion beam momentum  $P_\pi$  ( $W^2 \approx 2m_N P_\pi$ ) for  $\Delta_T^2 = 0$ .

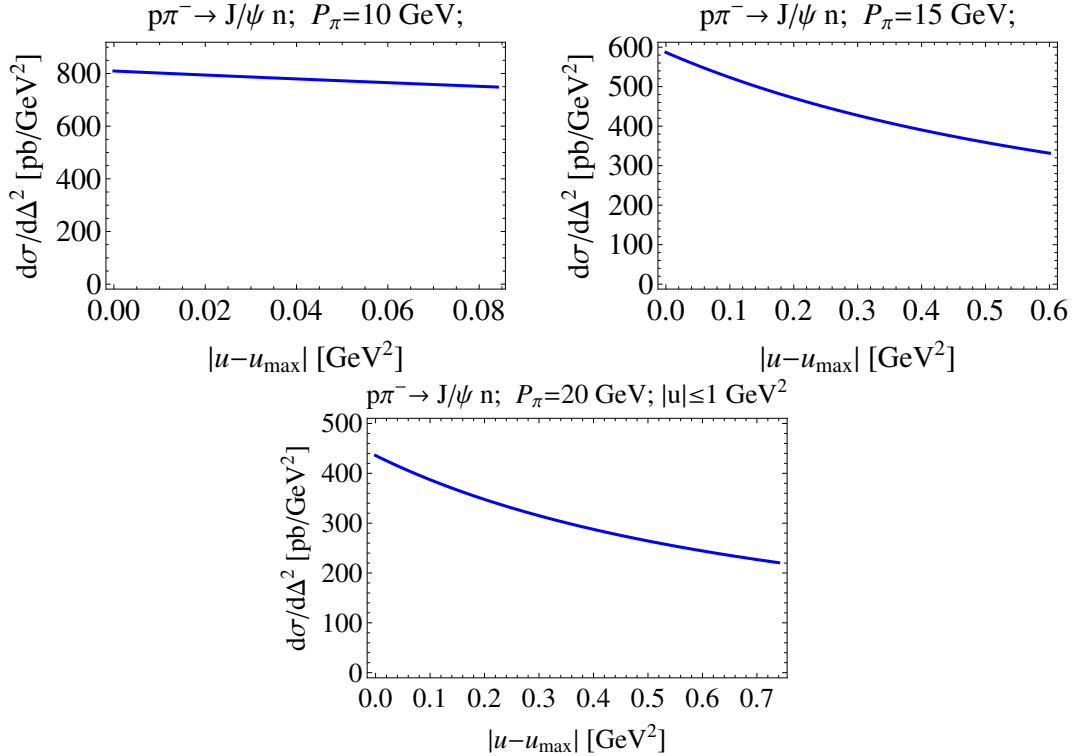


Figure 3: Differential cross section  $\frac{d\sigma}{d\Delta^2}$  for  $\pi^- p \rightarrow J/\psi n$  as a function of  $|u - u_{\max}|$  for  $P_\pi = 10, 15, 20$  GeV for  $|u| \leq 1$  GeV<sup>2</sup>.

On Fig. 4 we show the characteristic center of mass angular distribution for the  $d\sigma/d\Delta^2$  cross section for the backward factorization regime visualized on the polar plot

with the polar angle being the pion CMS scattering angle  $\theta_\pi^*$  (see eq. (10) for the definition). We present the ratio

$$\frac{\frac{d\sigma}{d\Delta^2}(W^2, \theta_\pi^*)}{\frac{d\sigma}{d\Delta^2}(W^2, \theta_\pi^* = 0)} \quad (24)$$

as a function of  $\theta_\pi^*$  showing the result for  $P_\pi = 10, 15, 20$  GeV and for  $-1 \text{ GeV}^2 \leq \Delta^2 \leq u_{\text{max}}$ , where  $u_{\text{max}}$  is the threshold value (12) of the momentum transfer squared. With the dashed lines we show the effect of the cutoff  $|\Delta^2| \leq 1 \text{ GeV}^2$  for the values of the CMS scattering angle  $\theta_\pi^*$  (see discussion around Eq. (10)).

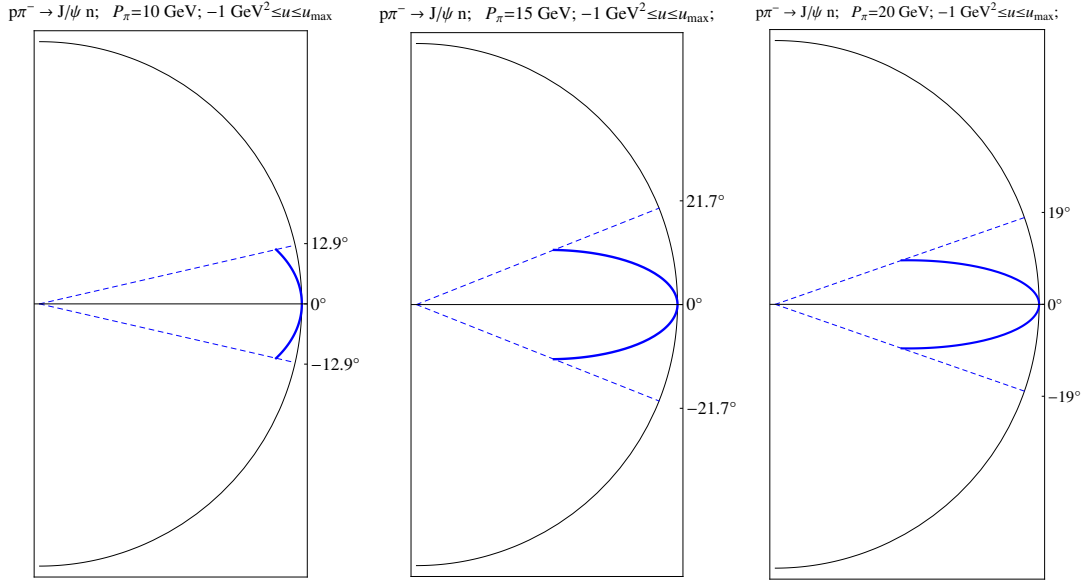


Figure 4: Angular distribution for the  $d\sigma/d\Delta^2$  cross section for the near-backward charmonium production ( $\theta_\pi^* \simeq 0$ ) for  $-1 \text{ GeV}^2 \leq \Delta^2 \leq u_{\text{max}}$ . Dashed lines show the effect of the cutoff  $|\Delta^2| \leq 1 \text{ GeV}^2$  for the values of the pion CMS scattering angle  $\theta_\pi^*$ .

Since these rates are certainly within the experimental reach of the J-Parc experiment, the study of the reaction (1) will provide a valuable universality test for the TDA approach since the same TDAs also arise in the description of  $N\bar{N} \rightarrow \gamma^*\pi$  [14],  $N\bar{N} \rightarrow J/\psi\pi$  [11, 12] and backward pion electroproduction off nucleon  $\gamma^*N \rightarrow \pi N$  [23].

## 5 Conclusions

In this paper we address the reaction  $\pi^- + N^p \rightarrow J/\psi + N^n$  which may be studied in the J-Parc facility. We argue that this reaction may be analyzed within the pQCD framework. It will not only help to quantitatively disentangle resonance production from the universal hadronic background but also will provide valuable information on hadronic structure encoded in pion-to-nucleon TDAs. Pion-to-nucleon TDAs supply

complementary information with respect to partonic distributions diagonal in quantum numbers such as common parton distributions and GPDs.

Within the kinematical range accessible at J-Parc we provide the predictions for the  $\pi^- + N^p \rightarrow J/\psi + N^n$  cross section using a simple nucleon pole model for  $\pi \rightarrow N$  TDAs. The obtained cross sections estimates give hope of experimental accessibility of the reaction. A specific feasibility study similar to that recently performed for accessing  $N \rightarrow \pi$  TDAs at PANDA [29, 11, 12] should be performed with J-Parc experimental efficiencies, as it has been done for exclusive forward lepton pair production at J-Parc [9].

It is worth mentioning that the mass of the charm quark may not be large enough for our leading order (in  $\alpha_s$ ) and leading twist analysis to be sufficient to describe the data. More work is certainly needed to go beyond the Born approximation for the hard amplitude, in particular because the timelike nature of the hard probe is often accompanied by large  $O(\alpha_s)$  corrections [30].

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## A Crossing $\pi \rightarrow N$ TDAs to $N \rightarrow \pi$ TDAs

We have the parametrization for the nucleon-to-pion ( $N \rightarrow \pi$ ) TDAs defined through the Fourier transform of the  $\pi N$  matrix element of the trilinear quark operator on the light cone. The parametrization involves eight invariant functions each being the function of three longitudinal momentum fractions, skewness variable, momentum transfer squared as well as of the factorization scale.

Throughout this paper we make use of the parametrization of Ref. [19], where only three invariant functions turn out to be relevant in the  $\Delta_T = 0$  limit. Let us consider the

neutron-to- $\pi^-$   $ud$  TDA

$$\begin{aligned}
& 4(p \cdot n)^3 \int \left[ \prod_{j=1}^3 \frac{d\lambda_j}{2\pi} \right] e^{i \sum_{k=1}^3 \tilde{x}_k \lambda_k (p \cdot n)} \langle \pi^-(p_\pi) | \varepsilon_{c_1 c_2 c_3} u_\rho^{c_1}(\lambda_1 n) u_\tau^{c_2}(\lambda_2 n) d_\chi^{c_3}(\lambda_3 n) | n(p_N, s_N) \rangle \\
&= \delta(\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3 - 2\tilde{\xi}) i \frac{f_N}{f_\pi} \left[ V_1^{(n \rightarrow \pi^-)}(\tilde{x}_{1,2,3}, \tilde{\xi}, \tilde{\Delta}^2) (\hat{p}C)_{\rho\tau} (U^+)_\chi \right. \\
&+ A_1^{(n \rightarrow \pi^-)}(\tilde{x}_{1,2,3}, \tilde{\xi}, \tilde{\Delta}^2) (\hat{p}\gamma^5 C)_{\rho\tau} (\gamma^5 U^+)_\chi + T_1^{(n \rightarrow \pi^-)}(\tilde{x}_{1,2,3}, \tilde{\xi}, \tilde{\Delta}^2) (\sigma_{p\mu} C)_{\rho\tau} (\gamma^\mu U^+)_\chi \\
&+ m_N^{-1} V_2^{(n \rightarrow \pi^-)}(\tilde{x}_{1,2,3}, \tilde{\xi}, \tilde{\Delta}^2) (\hat{p}C)_{\rho\tau} (\hat{\Delta}_T U^+)_\chi + m_N^{-1} A_2^{(n \rightarrow \pi^-)}(\tilde{x}_{1,2,3}, \tilde{\xi}, \tilde{\Delta}^2) (\hat{p}\gamma^5 C)_{\rho\tau} (\gamma^5 \hat{\Delta}_T U^+)_\chi \\
&+ m_N^{-1} T_2^{(n \rightarrow \pi^-)}(\tilde{x}_{1,2,3}, \tilde{\xi}, \tilde{\Delta}^2) (\sigma_{p\hat{\Delta}_T} C)_{\rho\tau} (U^+)_\chi + m_N^{-1} T_3^{(n \rightarrow \pi^-)}(\tilde{x}_{1,2,3}, \tilde{\xi}, \tilde{\Delta}^2) (\sigma_{p\mu} C)_{\rho\tau} (\sigma^{\mu\hat{\Delta}_T} U^+)_\chi \\
&\left. + m_N^{-2} T_4^{(n \rightarrow \pi^-)}(\tilde{x}_{1,2,3}, \tilde{\xi}, \tilde{\Delta}^2) (\sigma_{p\hat{\Delta}_T} C)_{\rho\tau} (\hat{\Delta}_T U^+)_\chi \right] \\
&\equiv \delta(\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3 - 2\tilde{\xi}) i \frac{f_N}{f_\pi} \sum_{\substack{\text{Dirac} \\ \text{structures}}} s_{\rho\tau, \chi}^{(N \rightarrow \pi)} H_s^{(n \rightarrow \pi^-)}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{\xi}, \tilde{\Delta}^2). \tag{A1}
\end{aligned}$$

We adopt Dirac's "hat" notation  $\hat{v} \equiv v_\mu \gamma^\mu$ ;  $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$ ;  $\sigma^{v\mu} \equiv v_\lambda \sigma^{\lambda\mu}$ ;  $C$  is the charge conjugation matrix and  $U^+ = \hat{p}\hat{n} U(p_N, s_N)$  is the large component of the nucleon spinor.  $f_\pi = 93$  MeV is the pion weak decay constant and  $f_N$  determines the value of the nucleon wave function at the origin.

Note that the  $N \rightarrow \pi$  TDA (A1) is defined with respect to its natural kinematical variables. Namely the cross channel momentum transfer is  $\tilde{\Delta} = p_\pi - p_N$  and the skewness parameter  $\tilde{\xi}$  is defined from the longitudinal momentum transfer between pion and nucleon

$$\tilde{\xi} \equiv -\frac{(p_\pi - p_N) \cdot n}{(p_\pi + p_N) \cdot n}$$

(i.e. it differs by the sign from the definition (4) natural for the reaction (1)).

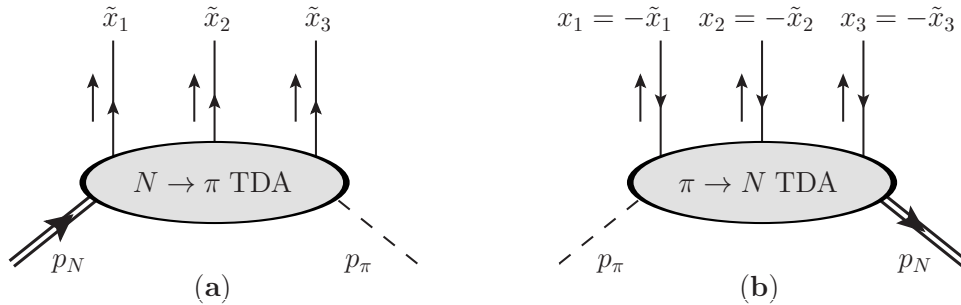


Figure 5: Small arrows show the direction of the longitudinal momentum flow in the ERBL-like regime for (a): The longitudinal momentum flow for  $N \rightarrow \pi$  TDAs defined in (A1). The longitudinal momentum transfer is  $(p_\pi - p_N) \cdot n \equiv \tilde{\Delta} \cdot n$ . (b): The longitudinal momentum flow for  $\pi \rightarrow N$  TDAs defined in (17). The longitudinal momentum transfer is  $(p_N - p_\pi) \cdot n \equiv \Delta \cdot n$ . Arrows on the nucleon and quark (antiquark) lines show the direction of flow of the baryonic charge.

Now we would like to express pion-to-nucleon ( $\pi \rightarrow N$ ) TDAs through ( $N \rightarrow \pi$ ) TDAs occurring in (A1). For this issue we apply the Dirac conjugation (complex conjugation and convolution with  $\gamma_0$  matrices in the appropriate spinor indices) for both sides of eq. (A1) and compare the result to the definition of  $\pi \rightarrow N$  TDAs (17):

$$\begin{aligned}
& -4(p \cdot n)^3 \int \left[ \prod_{j=1}^3 \frac{d\lambda_j}{2\pi} \right] e^{-i \sum_{k=1}^3 \tilde{x}_k \lambda_k (p \cdot n)} \langle n(p_N, s_N) | \varepsilon_{c_1 c_2 c_3} \bar{u}_\rho^{c_1}(\lambda_1 n) \bar{u}_\tau^{c_2}(\lambda_2 n) \bar{d}_\chi^{c_3}(\lambda_3 n) | \pi^-(p_\pi) \rangle \\
& = -\delta(\tilde{x}_1 + \tilde{x}_2 + \tilde{x}_3 - 2\tilde{\xi}) i \frac{f_N}{f_\pi} \sum_s \underbrace{(\gamma_0^T)_{\tau\tau'} \left[ s_{\rho'\tau',\chi'}^{(N \rightarrow \pi)} \right]^\dagger (\gamma_0)_{\rho'\rho} (\gamma_0)_{\chi'\chi}}_{s_{\rho\tau,\chi}^{(\pi \rightarrow N)}} H_s^{(N \rightarrow \pi)}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{\xi}, \Delta^2).
\end{aligned} \tag{A2}$$

For the relevant Dirac structures we get

$$\begin{aligned}
(v_1^{(\pi \rightarrow N)})_{\rho\tau,\chi} &= (C\hat{p})_{\rho\tau} \bar{U}_\chi^+; \\
(a_1^{(\pi \rightarrow N)})_{\rho\tau,\chi} &= (C\hat{p}\gamma_5)_{\rho\tau} (\bar{U}^+ \gamma_5)_\chi; \\
(t_1^{(\pi \rightarrow N)})_{\rho\tau,\chi} &= -(C\sigma_{p\mu})_{\rho\tau} (\bar{U}^+ \gamma_\mu)_\chi; \\
(v_2^{(\pi \rightarrow N)})_{\rho\tau,\chi} &= (C\hat{p})_{\rho\tau} (\hat{\Delta}_T \bar{U}^+)_\chi = -(C\hat{p})_{\rho\tau} (\hat{\Delta}_T \bar{U}^+)_\chi; \\
(a_2^{(\pi \rightarrow N)})_{\rho\tau,\chi} &= (C\hat{p}\gamma_5)_{\rho\tau} (\bar{U}^+ \hat{\Delta}_T \gamma_5)_\chi = -(C\hat{p}\gamma_5)_{\rho\tau} (\bar{U}^+ \hat{\Delta}_T \gamma_5)_\chi; \\
(t_2^{(\pi \rightarrow N)})_{\rho\tau,\chi} &= -(C\sigma_{p\hat{\Delta}_T})_{\rho\tau} (\bar{U}^+)_\chi = (C\sigma_{p\Delta_T})_{\rho\tau} (\bar{U}^+)_\chi; \\
(t_3^{(\pi \rightarrow N)})_{\rho\tau,\chi} &= (C\sigma_{p\mu})_{\rho\tau} (\bar{U}^+ \sigma_{\mu\hat{\Delta}_T})_\chi = -(C\sigma_{p\mu})_{\rho\tau} (\bar{U}^+ \sigma_{\mu\Delta_T})_\chi; \\
(t_4^{(\pi \rightarrow N)})_{\rho\tau,\chi} &= -(C\sigma_{p\hat{\Delta}_T})_{\rho\tau} (\bar{U}^+ \hat{\Delta}_T)_\chi = -(C\sigma_{p\Delta_T})_{\rho\tau} (\bar{U}^+ \hat{\Delta}_T)_\chi,
\end{aligned} \tag{A3}$$

where we switch to the definition of momentum transfer natural for the reaction (1):  $\tilde{\Delta} \rightarrow -\Delta$ .  $\bar{U}^+ \equiv \bar{U}(p_N) \hat{n} \hat{p}$  stands for the large component of the  $\bar{U}(p_N)$  Dirac spinor.

The flow of the longitudinal momentum for  $N \rightarrow \pi$  TDAs defined as in eq. (A1) and  $\pi \rightarrow N$  TDAs defined as in eq. (17) is presented on Fig. 5. By switching to the variables  $\xi = -\tilde{\xi}$  and  $x_i = -\tilde{x}_i$  natural for the reaction (1) and  $\tilde{\Delta}^2 \rightarrow \Delta^2$  and comparing (A2) to (17) we conclude that

$$\begin{aligned}
& \left\{ V_{1,2}^{(\pi^- \rightarrow n)}, A_{1,2}^{(\pi^- \rightarrow n)}, T_{1,2,3,4}^{(\pi^- \rightarrow n)} \right\} (x_{1,2,3}, \xi, \Delta^2) \\
& = \left\{ V_{1,2}^{(n \rightarrow \pi^-)}, A_{1,2}^{(n \rightarrow \pi^-)}, T_{1,2,3,4}^{(n \rightarrow \pi^-)} \right\} (-x_{1,2,3}, -\xi, \Delta^2).
\end{aligned} \tag{A4}$$

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